

DEMOGRAPHIC SHOCKS AND THE FACTOR PROPORTIONS MODEL: FROM THE PLAGUE OF JUSTINIAN TO THE BLACK DEATH

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All of his professional life Eli Heckscher was concerned with the methodology of economics and economic history. Straddling both disciplines it was essential for him to come to grips with the problem of what one discipline could learn from the other and vice versa (Findlay, 1998, Findlay and Lundahl, 2002, Henriksson and Lundahl, 2003). Gradually he also introduced the use of statistical time series in his works, notably his four-volume *magnum opus* about the economic history of Sweden from the time of Gustav Vasa (1523-1560) to ‘the present’, which in practice meant the early nineteenth century. As he moved from the sixteenth to the eighteenth century the availability of quantitative material increased, and Heckscher made use of it. However, he steadfastly refused to be bound by the strict limits imposed by the ‘hard’ facts when it came to the interpretation of a certain epoch. He certainly took great care to weed out hypotheses not grounded in facts, but he was no stranger to hypothetical reasoning either. Such reasoning was needed for arriving at a historical synthesis, and the refusal to go beyond just what the sources, after thorough critical scrutiny, would reveal was for him to stop short of attempting a synthesis.

The present essay should be seen as the effort of two economists to provide a building block for a historical synthesis. We would like to combine Heckscher’s plea for economic theory in economic history with his insistence on historical synthesis into a plea for economic theory in historical synthesis. The roots of the essay are found in economic theory, which we employ very much in the fashion that Heckscher used it in his historical works: as a device for explaining the equilibrating processes in economic history during a determined period. We also share with Heckscher the conviction that it is difficult to use economic theory to explain transitions from one historical epoch or era to another. Here factors exogenous from the point of view of economics must be invoked, and we begin and end our story with two such events: the so-called Plague of Justinian in the mid-sixth century A.D. and the Black Death eight hundred years later.

What our analysis deals with is precisely the period between these two events, and the synthesis that concerns us is one based on a marriage between the historical facts, as provided by the professional historians, and an economic theory that may serve as reading glasses when it comes to the interpretation of the events. Our aim is to provide a reading of some of the fundamentals of eight centuries of the history of western Europe, eastern Europe and Islam that is consistent with the interplay of the probable basic economic mechanisms at work, i.e. we want to put history on a sound theoretical footing, to heed Heckscher's (1929) plea for the use of theory in economic history, and simultaneously to allow economic history to influence theorizing. Heckscher made it clear that theory was useful when it came to organizing the questions to be put to the historical material, and we agree, but any old theory won't do. In order to be efficient, our theory must spring out of the historical material itself. Its categories must be chosen in a way that properly reflects the characteristics of the period we have chosen to deal with. It is only through the interaction between the given historical material and the specific tools we need to construct to reveal its hidden secrets that we can arrive at a better understanding of the underlying processes that made economic history take a particular turn at given moment in time.

The Plague of Justinian and its Aftermath

The waves of bubonic plague that swept the world of Late Antiquity and the Early Middle Ages have taken their name from the Emperor Justinian I, ruler of the eastern Roman Empire from 527 to 565. It was during his reign that the first outbreak occurred in Constantinople in 542, killing hundreds of thousands of people according to the historian Procopius, secretary to Justinian's general Belisarius. Procopius, who was an eyewitness, has provided us with the first incontestable description of bubonic plague in history (quoted by Bray, 2000:22-23):

During these times there was a pestilence, by which the whole human race came near to being annihilated, ... it did not come in a part of the world, nor upset certain men, nor did it confine itself to any season of the year ... but it embraced the whole of the world, and blighted the lives of all men, though differing one from another in the most marked degree, respecting neither sex nor age ...

[...] with the majority it came about that they were seized by the disease without becoming aware of what was coming. They had a certain fever ... The

body showed no change from its previous colour, nor was it hot ... nor did any inflammation set in ... But on the same day, in some cases, in others on the following day, and in the rest not many days after, a bubonic swelling developed; and this took place not only in the particular part of the body which is called the 'boubon', that is below the abdomen, but also inside the armpit, and in some cases also beside the ears and at different points on the thigh For there ensued with some a deep coma, with others a violent delirium ... for neither physicians nor other persons were found to contract this malady through contact with the sick or with the dead, for many were constantly engaged in burying or attending those in no way connected with them. ... With some the body broke out in black pustules about as large as a lentil and these did not survive even one day, but all succumbed immediately. With many also a vomiting of blood ensued without visible cause and straightaway brought death ... In some cases when the swelling rose to an unusual size and discharge of pus had set in, it came about that they escaped from the disease and survived.

The emperor himself was infected, but survived. The first wave of the 540s-550s was followed by several others of varying intensity, persisting intermittently until the middle of the eighth century (Biraben and LeGoff, 1969). While there is wide disagreement as to the extent of the impact (cf. e.g. Harrison, 1999:141:53, for a critical view), there is little doubt that it was a demographic catastrophe on a scale not exceeded till the Black Death of the fourteenth century. Russell (1968:180) estimates a 20-25 percent loss for the first epidemic of 541-544 and a total loss of 40-50 percent of the pre-plague population over the period 540-700.

The plague is believed to have entered the Mediterranean world at the Egyptian port of Pelusium through Ethiopia and the Red Sea, before spreading both east and west. The populations of Egypt, Syria and Palestine, then all under the sway of the Byzantine Empire, were severely affected. What happened further east is not clear (Bray, 2000:27):

Its distribution to the east of Syria is something of a puzzle. It would seem that the Arabs did not bring the plague back to the Hejaz ... Equally it would seem that the Arabs did not bring the plague to Afghanistan, the Indus, Ferghana and Transcaucasia and thus into the Indian subcontinent and China, as all observers have the plague ceasing at about the present western border of Iran and at the Caucasus. Whether it penetrated Africa south of Egypt is anyone's guess.

The Middle East as well was hit by successive outbreaks of the plague. Dols (1974) lists six 'major' epidemics between 627 and 717. The westward spread to Italy, Gaul and Spain was through the seaports of Genoa, Marseilles and Narbonne, proceeding

inland along the rivers and trade routes. The impact on the northern lands seems to have been much less severe, though not necessarily negligible. The arrival of the plague at Marseilles in 588 was vividly described as follows by the contemporary Gallo-Roman historian Gregory of Tours (1974:510-511):

...a ship from Spain put into port with the usual kind of cargo, unfortunately also bringing with it the source of the infection. Quite a few of the townsfolk purchased objects from the cargo and in less than no time a house in which eight people lived was left completely deserted, all the inhabitants having caught the disease. The infection did not spread through the residential quarter immediately. Some time passed, and then, like a cornfield set alight, the entire town was suddenly set ablaze with the pestilence ... at the end of two months the plague burned itself out. The population returned to Marseilles, thinking themselves safe. Then the disease started again and all who had come back died. On several occasions later on Marseilles suffered from an epidemic of this sort.

Before the plague struck, the eastern Roman Empire under Justinian was at the height of its power, with plentiful resources of manpower and revenue. Not only was he able to secure the eastern frontier against the Persians but also to stabilize and push back the frontiers against the Avars, Lombards, Berbers and others, while soundly defeating the Vandals in North Africa in 534-35. The great scheme of once again unifying the eastern and western halves of the empire was not impossible, though of course extremely difficult. The devastating effects of the plague on both manpower and revenue, however, rendered it impossible. Nevertheless, although his plans for expansion failed he was still able to retain most of the eastern territories of the empire.

As often noted, the nomads of the Arabian Peninsula escaped the ravages that the plague wrought on the more settled Byzantine and Sassanid empires. After the unification of the tribes under the Prophet and his early successors, the 'rightly guided' Caliphs, at the beginning of the seventh century, the Arabs rapidly captured Syria, Palestine, Egypt, North Africa and western Mesopotamia from the Byzantines. Sassanid Iran was conquered later by the Arabs under the Ummayyad Caliphate. The Byzantine emperor Heraclius (610-641) and his successors could not match the élan of the Arab onslaught with the depleted resources at their command. They retreated to the Anatolian plateau behind the security of the Taurus mountain range and then successfully resisted successive Arab attacks on Constantinople from the sea.

The empire was to survive for eight hundred more years but it had lost the bulk of its territory, population and revenue to the Arabs. In the west the empire's hold on Italy was rendered extremely tenuous, and the Lombard invasion of the peninsula, beginning in 568, could not be checked before most of it had been occupied. Gaul was left to the Merovingian Franks. Spain, under Visigothic rulers, was invaded in by the Muslims in 711, who established the powerful Emirate of Cordoba that occupied most of the Iberian Peninsula. The popes in Rome ceased to look to Constantinople and eventually allied themselves with the rising dynasty of the Frankish warlord Charles Martel, who checked the Arab advance at Poitiers in 732 or 733.¹ Around the same time, say 718 or 722 (Collins, 1995:182), after a Christian victory over Muslim forces at Covadonga in Asturias, the Spanish *Reconquista* began.

By the second half of the eighth century the territory of the Roman Empire at its height had come to be divided between three great powers, the Greek Orthodox Byzantine Empire under the vigorous new Isaurian Dynasty, with its capital at Constantinople, the Muslim Abbassid Empire, with its capital at Baghdad and the Latin Catholic Carolingian Empire with its capital at Aachen. Beyond the frontiers of these empires were Anglo-Saxon England in the west, the pagan Scandinavian and Saxon tribes in the north and in the east the Slavs and Bulgars in the Balkans.

Population figures for such early periods in history are notoriously unreliable and can only be taken as the best guesses made by scholars on the basis of extremely scanty evidence. Issawi (1981) citing Russell (1968) gives figures of 16.6 million in 350 for the eastern Roman Empire, falling to 10 million by 600, indicating the devastating effect of the early waves of the Plague of Justinian, Treadgold (1997:278) gives a figure of as much as 26 million for the empire under Justinian in 540, before the plague, falling to 17 million in 610 before the Arab invasions and collapsing to 7 million in 780 after that disaster, rising back to 12 million by 1025. Issawi cites 15 million at least for 1000, or about 25 percent more than Treadgold. The general qualitative picture however is similar, with a heavy loss of population due to plague, more loss due to the Arab invasions, and then recovery within the restricted territory.

¹ The traditional date is October 732, but it could also be October 733 (Fouracre, 2000:87).

Issawi's main contribution is an estimate of the area and population of the Arab Empire at its height in 750. He puts the population at between 28 and 36.5 million over the vast arc stretching from Spain to India. The breakdown is 10-13 million in the former Byzantine territories of Egypt, Syria and North Africa; 5-6 million in Spain; 1.5-2 million in the Arabian peninsula; 5-6 million in Iraq; 3-4 million in Iran and 3.5-5.5 million in Central Asia, Afghanistan and India. The gross area he puts at 9.8 million square kilometers and the inhabited area at 2.1 million, only slightly more than one-fifth, indicated the highly arid inhospitable nature of much of the terrain.

For the 10th to 11th centuries Issawi estimates only a very modest rise of the population to 35-40 million, or about 25 percent over three centuries. The Abbassid Empire was about twice as big in both population and inhabited area as the contemporary Byzantine Empire. The only comparable source of population history for the world as a whole is McEvedy and Jones (1980). We have checked Issawi's figures against the comparable estimates that they provide. While their figures are generally lower they are not too far below the bottom range of Issawi. The figures based on their estimates of individual countries are 21 million for 700, 26 million for 1000 and 22 million for 1300. Thus the trend of population for the Islamic world is a modest 25 percent over three centuries for both Issawi and McEvedy and Jones. The fall from 26 million in 1000 to 22 million in 1300 is attributable to the ravages of the Mongol invasion in Iran and Iraq in the east and the Bedouin raids in North Africa in the west.

For Europe excluding Russia McEvedy and Jones report a peak of 33 million in 200, falling to a trough of 23 million in 600, rising back to 32 million by 1000 and then accelerating upwards to the huge figure of 70 million in 1300 on the eve of the Black Death.² This gives annual growth rates of less than 0.1 percent per annum for the earlier period and 0.25-0.3 percent after the turn of the millennium. David Grigg (1980:53) reports some figures for individual countries: 0.43 percent per annum for England and Wales from 1086 to 1340, 0.49 percent for France from 1100 to 1328, less than 0.2 percent for Italy from 950 to 1300 and for Denmark from 1000 to 1300,

² Livi-Bacci (2001:27), quoting Biraben (1979:16), has similar figures: 200: 44 million, 600:22 million, and 1340: 74 million.

0.46 percent for the Moselle valley between 1000 and 1237, and an average rate of increase for Europe between 1000 and 1340 of 0.26 percent.

While these figures are notoriously uncertain, representing a tendency only, the beginnings of the higher demographic growth that was to last for three centuries can be dated. 'It was 930-50 in Sabina and Lombardy, 940-90 in Catalonia, 980-1010 in Languedoc, Provence, Poitou and the Auvergne, 1010-30 in Flanders and Picardy, Bavaria and Franconia, Burgundy and Normandy, 1050-80 in England and the Rhineland, after 1100 in central Germany' (Fossier, 1999:62). If we are to believe the McEvedy-Jones estimates, the increase from the trough in 600 to the peak in 1300 is a remarkable threefold or 200 percent over the seven centuries. For comparison, China increased from the 200 peak of 60 million back to 60 million in 1000, just like Europe, before rising further to a peak of 115 million in 1200, before falling to 85 million in 1300 as a result of the Mongol invasions at the other end of the European peninsula. This approximate doubling took place under the Sung Dynasty, generally regarded as the 'golden age' of imperial China. Yet it does not come close to the European achievement.

Why did the western Europe of the so-called dark ages perform so spectacularly well, relative to what were at that time the much more advanced Byzantine, Islamic and Sung Chinese civilizations? Clearly, the foregoing leads us to suspect that the demographic change caused by the plague had something to do with it.³ As it seems, the economic trend of late antiquity was a downward one in the Mediterranean area. In his monumental work on the 'dark' centuries of the European economy, Michael McCormick (2001:41) concludes his chapter of the end of the ancient world thus:

Settlement patterns suggest that ill-understood processes of demographic stagnation and decline moved slowly across the old Roman space. They finally reached the east in the sixth or early seventh century. Around the same time, the history of disease marks a new configuration in the health experience of the population. The new pathocoenosis [the array of diseases characteristic of a society] could not by itself have been the leading cause of wide-reaching economic change, for change had started earlier in most of the Mediterranean world. But it surely must have reinforced some aspects of that change, if only

³ The plague may not have been the only disease to gain a strong foothold in late antiquity. As McCormick (2001:38-41), has pointed out, both malaria and leprosy appear to have been on the rise in the Roman world at about the same time as when the plague of Justinian struck.

by debilitating or destroying part of the work force. Both settlement patterns and disease encourage us to believe that the overall trend of the late Roman world was downward between c. 200 and 700.

The next section of this paper will develop a model of demographic-economic-ecological interaction that could perhaps provide some clues to the very fundamental problem of the unequal patterns of decline and, later, rebirth.

The Model

The model presented in this section draws on our previous work (Findlay, 1993, Findlay and Lundahl, 2002) combining the Malthusian demographic specification with the concept of an endogenous land frontier, while stripping away the other features of those models.

The Production Function

The economy is considered as producing a single input, denoted Y , with land, A and labor, L , as inputs, according to the production function

$$Y = Y(A,L) \tag{1}$$

which is taken to have the usual neoclassical properties of constant returns to scale, positive first and negative second derivatives with respect to each input and complementarity between the inputs. Constancy of returns to scale enables us to write

$$y = y(a) \tag{2}$$

where y and a denote Y and A divided by L , respectively. We can also write the rent per acre and the real wage as

$$r = y'(a) \tag{3}$$

$$w = y(a) - y'(a)a \tag{4}$$

The Malthusian Mechanism

The fertility and mortality rates of the population are specified as positive and negative functions, respectively, of the per capita *consumption*, c , of the population

$$f = f(c), \quad f'(c) > 0 \quad (5)$$

$$m = m(c), \quad m'(c) < 0 \quad (6)$$

These relations are depicted in *Figure 1* below.

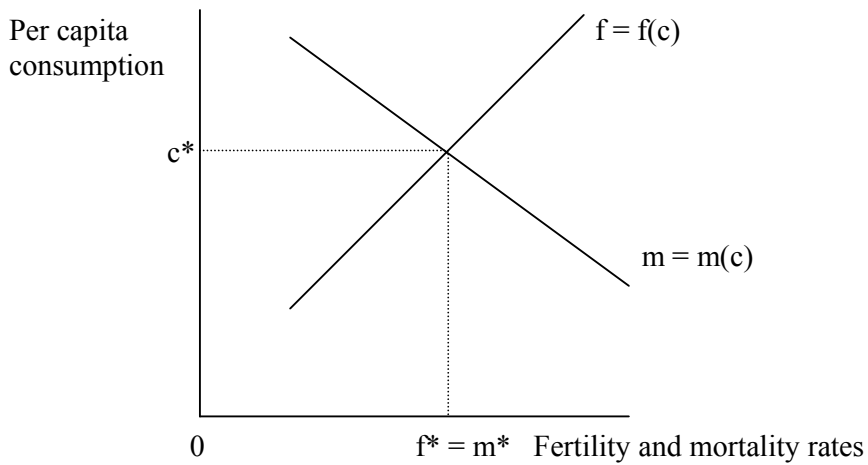


Figure 1: The Malthusian mechanism

The population is in equilibrium at the per capita consumption level of c^* that equates $f(c^*)$ and $m(c^*)$ at f^* and m^* .

Labor Productivity and Per Capita Consumption

Labor productivity, as defined above, is

$$y = Y/L \quad (7)$$

Suppose that we are in a 'stationary state' with zero net investment. Total consumption will then be equal to total output and the per capita consumption of the population as a whole will be

$$c = Y/P \tag{8}$$

where P denotes the entire population.

Suppose that the labor force is proportional to the population so that

$$L = \alpha P \tag{9}$$

where α is some fraction. In the pre-industrial economic conditions that we are considering, the non-working population will consist of warriors priests another ‘unproductive’ occupations as well as those too old or too young to work. Taking a more ‘physiocratic’ view, we could even include in the $(1 - \alpha)P$ artisans, traders and so on who are ‘supported’ by the surplus generated by agriculture, the only truly ‘productive’ sector from this standpoint. From equations (7) to (9) it follows that

$$y^* = c^*/\alpha \tag{10}$$

so that y^* is the level of labor productivity that will maintain output, population and the labor force of the ‘stationary state’ levels compatible with the Malthusian equilibrium level of per capita consumption, c^* .

From (2) we can obtain the value a^* of the land-labor ratio that defines y^* as

$$y^* = y(a^*) \tag{11}$$

We have therefore determined the long-run stationary volumes of the intensive magnitudes c^* , y^* and a^* , but we yet have to determine the absolute levels of Y , C , A and P .

Land and the Frontier

The marginal productivity of land and hence the rent per acre is determined in the stationary state as

$$r^* = y'(a^*) \quad (12)$$

We assume that an acre of land deteriorates in fertility at a rate of μ unless it is maintained. Suppose that the constant rate of time preference in the economy, and hence the rate of interest, is equal to δ .

Then by the usual asset pricing formula, the price of an acre of land would be

$$p^* = y'(a^*)/(\delta + \mu) \quad (13)$$

Figure 2 depicts the determination of p^* by the intersection of the demand and supply curves for the stock of land in the stationary state.

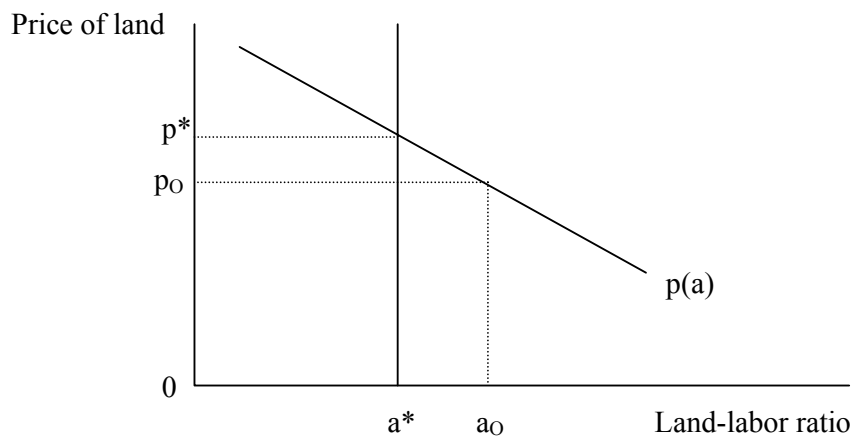


Figure 2: Stock equilibrium in the land market

The downward-sloping curve $p(a)$ in *Figure 2* shows the demand price for an acre of land as a function of the land-labor ratio. Since

$$y''(a) < 0$$

by the 'diminishing returns' property of the production function (2), the volume of $y'(a)$ capitalized by the reciprocal of $(\delta + \mu)$ is the negatively shaped curve $p(a)$ in *Figure 2*. The intersection with the vertical supply curve of land per unit of labor a^* in the stationary state yields p^* as in (13).

We now turn to the determination of the actual size of the land area A^* of the economy in the stationary state. Once this is determined, of course, the levels of L^* , Y^* , P^* and C^* can all be obtained as well since we know the equilibrium intensive magnitudes a^* , y^* and c^* .

The area of arable land A^* ultimately available in an economic system depends upon climatic or geographical factors, on the one hand, and the technology of land clearance and maintenance, on the other. Population, and hence the labor force, is endogenously determined within a Malthusian framework by the same factors and hence is not an independent variable in its own right.

In *Figure 3* below we consider the arable land available, A , as a function of the labor L_a , that is needed to ‘establish the frontier’, i.e. to ensure that A acres are available to the economy. To express it differently, L_a is the *cumulative* amount of labor that has gone into the clearance of the A acres of land currently available in the economy. We postulate that

$$L_a = L_a(A), \quad L_a'(A) > 0, \quad L_a'' > 0 \quad (14)$$

where $L_a'(A)$ is the marginal labor cost of clearing an additional acre, which increases with the amount of land already cleared, as indicated by the second derivative being positive as well. This formulation illustrates the Ricardian idea that extending the margin of cultivation is an increasingly costly enterprise. The convex function (14) is plotted as the convex graph of $L_a(A)$ in *Figure 3*.

In the long-run stationary equilibrium it will be true that

$$w^*L_a'(A) = p^* \quad (15)$$

i.e. that the real marginal cost of clearing an additional acre, the left-hand side of (15), must be equal to the equilibrium price per acre p^* . Note that w^* and p^* in (15) have already been obtained by (4) and (13) alone. This enables us to solve (15) for the unique value A^* of A that satisfies this equation. This solution is depicted graphically

in *Figure 3* where the slope of the convex function $L_a(A)$ at A^* is the value of $L_a'(A)$ that satisfies (15), given w^* and p^* as determined already.

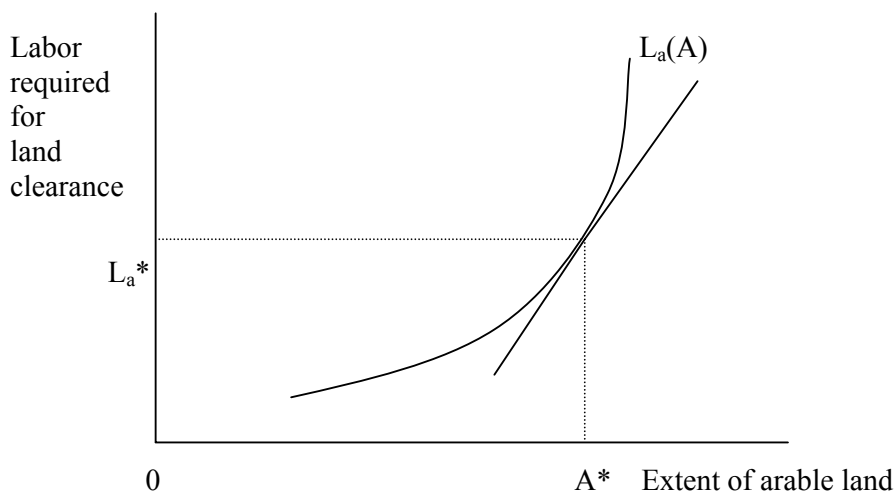


Figure 3: Determination of the land frontier

It is instructive to substitute for p^* from (13) into (15) to obtain the relation

$$w^* = y'(a)[1/L_a'(A)][1/(\delta + \mu)] \quad (16)$$

The right-hand side of (16) can be interpreted as the ‘indirect’ marginal productivity of labor in clearing additional land, the marginal productivity $y'(a)$ of which is capitalized by the ‘gross’ discount factor, the reciprocal of $(\delta + \mu)$. Since w^* is also equal to its direct marginal product by (4) we have the ‘efficiency condition’ for labor allocation that it requires its direct and indirect marginal productivities to be equal.

Having now determined the absolute magnitude A^* of arable land and knowing already the equilibrium values a^* , y^* and c^* of the intensive magnitudes, we can easily obtain the absolute values L^* , Y^* , P^* and C^* of the labor force, total output, population and consumption. We have thus fully determined the equilibrium values of all the variables in the model.

The relation between Y , A and L is depicted in *Figure 4*. Given A^* the concave function shows the relation between Y and L determined by the production function (1). The ray from the origin has a slope equal to y^* , so that the intersection with the concave function determines L^* and Y^* .

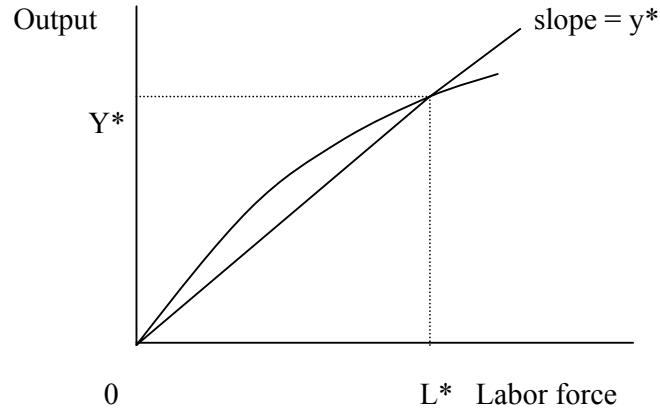


Figure 4: Determination of total output

Employment in Land Clearance

Above we established the cumulative labor effort needed to provide the economy with an arable land area of A^* . However, since land is assumed to ‘depreciate’ at the rate μ , for example through reforestation or soil erosion, it must continuously be replenished by the clearance of *new* land for the stationary state to be maintained. The amount of new land cleared will be an increasing function of the price of land and a decreasing function of the real wage, equal to p^* and w^* , respectively, in the stationary state. Letting N denote the amount of *new* land cleared (as opposed to the cumulative total, A) and L_n the amount of current labor required for this task, in the stationary state we must have

$$N(L_n) = \mu A^* \quad (17)$$

and

$$L_n^* = L_a(A^*) - L_a[(1 - \mu)A^*] \quad (18)$$

Assuming land clearance to be an explicit economic activity, ‘profits’ equal to $(p^*N - w^*L_n)$ must be maximized, which requires that

$$P^*(\partial N/\partial L_n) = w^* \quad (19)$$

which is precisely the condition (15) obtained earlier to determine A^* by equating the price p^* to the marginal cost of land clearance $w^*L_a'(A)$. (At the margin it does not matter whether we use A or N .)

Where does L_N^* come from? The number of production workers L is equal to αP by assumption, so the L_N^* workers required for the land clearance come out of the remaining pool of $(1 - \alpha)P$. Thus the labor force is the sum of the production workers and the members of the ‘land-clearing brigade’, and if we add the ‘unproductive’ members of society we get the total population.

What happens outside of the steady state? Corresponding to any point (A, L) in the input space the production function determines $r(a)$ and $w(a)$ by the marginal productivities of each input. Capitalizing $r(a)$ by the reciprocal of $(\delta + \mu)$ and ignoring expected capital gains and losses we can always obtain $p(a)$ for every $r(a)$. Taking $p(a)$ and $w(a)$ as given the land-clearing sector maximizes profits in determining the amount of land cleared $N(p, w)$ as an increasing function of p and a decreasing function of w as indicated by

$$N = N(p, w) \quad \partial N/\partial p > 0, \quad \partial N/\partial w < 0 \quad (20)$$

Since the point (A, L) is arbitrary it is clear that $N(p, w)$ may exceed, equal or fall short of μA , allowing A to rise or fall over time depending upon the direction of the inequality.

Dynamic Stability

From a dynamic perspective the model can be represented by the following two-dimensional system of differential equations

$$\dot{L} = \dot{L}(L, A) \quad (21)$$

$$\dot{A} = \dot{A}(L, A) \quad (22)$$

where the dots indicate the time derivatives dL/dt and dA/dt respectively.

The equilibrium state of this system is reached when the values of L and A are such that (21) and (22) are both equal to zero. These values are precisely the L^* and A^* that we have obtained above for the long-run stationary state. What is investigated here is whether the system will move towards the point (L^*, A^*) when $L(t)$ and $A(t)$, the state variables of the system are not equal to these equilibrium values.

The system will be dynamically stable, i.e. $L(t)$ and $A(t)$ will approach (L^*, A^*) , when the Jacobian matrix of the partial derivatives of (22) and (23)

$$J \equiv \begin{vmatrix} \dot{\partial L/\partial L} & \dot{\partial L/\partial A} \\ \dot{\partial A/\partial L} & \dot{\partial A/\partial A} \end{vmatrix}$$

evaluated at the equilibrium point (L^*, A^*) satisfies

$$\dot{\partial L/\partial L} + \dot{\partial A/\partial A} < 0 \quad (23)$$

$$(\dot{\partial L/\partial L})(\dot{\partial A/\partial A}) - (\dot{\partial A/\partial L})(\dot{\partial L/\partial A}) > 0 \quad (24)$$

The first of these is called the ‘trace’ and the second the ‘determinant’ condition. Together they ensure that the characteristic roots of the matrix are negative, and have negative real parts, as required for dynamic stability.

We may now evaluate each of the four partial derivatives to obtain their signs.

The rate of change of population, and hence the labor force, is given by the difference between fertility and mortality rates:

$$\dot{L} = [f(c) - m(c)]L \quad (25)$$

from equations (5) and (6) above. Noting that per capita consumption c is proportional to per capita output y as given by (10) we obtain the sign of the partial derivative

$$\dot{\partial L / \partial L} = [f'(c) - m'(c)]\alpha(\partial y / \partial L) < 0 \quad (26)$$

Since $f'(c)$ is positive and $m'(c)$ is negative the expression within parenthesis is positive. But

$$\partial y / \partial L = y'(a)(-A/L^2) < 0 \quad (27)$$

since average productivity per worker falls when L is increased with A constant, ensuring (26).

Partially differentiating (25) with respect to A we obtain

$$\dot{\partial L / \partial A} = [f'(c) - m'(c)]\alpha y'(a) > 0 \quad (28)$$

From the previous section we have seen that

$$\dot{A} = N(p, w) - \mu A \quad (29)$$

Both p and w are functions of A and L , so we have

$$\dot{\partial A / \partial A} = (\partial N / \partial p)(\partial p / \partial A) + (\partial N / \partial w)(\partial w / \partial A) - \mu < 0 \quad (30)$$

since

$$\partial N / \partial p > 0, \quad \partial p / \partial A < 0, \quad \partial N / \partial w < 0, \quad \partial w / \partial A > 0$$

and

$$\dot{\partial A / \partial L} = (\partial N / \partial p)(\partial p / \partial L) + (\partial N / \partial w)(\partial w / \partial A) > 0 \quad (31)$$

since

$$\partial p / \partial L > 0, \quad \partial w / \partial L < 0$$

The 'trace' condition (23) is clearly satisfied, since both diagonal elements of the matrix J have been shown to be negative.

Figure 5 below is the 'phase diagram' of the dynamic system (21)–(22). It shows combinations of the state variables A and L that leave $dL/dt = 0$ and $dA/dt = 0$ along each of the respective functions, with the equilibrium point being (A^*, L^*) where these two functions intersect. It is easy to see that the 'determinant' condition (24) is satisfied if the slope of the $dL/dt = 0$ function is steeper than the slope of the $dA/dt = 0$ function, i.e.

$$(\partial \dot{L} / \partial L) / (\partial \dot{L} / \partial A) > (\partial \dot{A} / \partial L) / (\partial \dot{A} / \partial A)$$

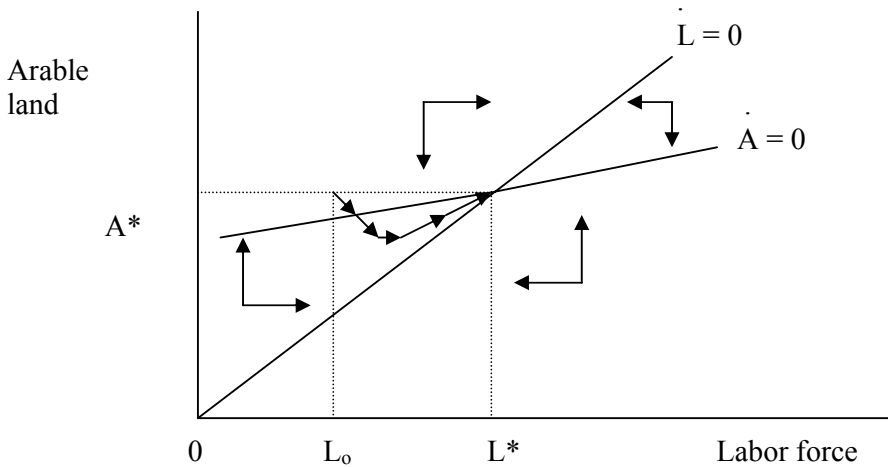


Figure 5: The dynamic stability of the model

The $dL/dt = 0$ function is a ray through the origin with a slope equal to a^* , the land-labor ratio that equates the fertility and mortality rates to maintain the Malthusian equilibrium. For any given value of A points to the left of the $dL/dt = 0$ ray have a greater than a^* and hence c greater than c^* , so population, and hence the labor force,

must increase. By the same reasoning L must increase for a given A from any point to the right of the $dL/dt = 0$ ray. Similarly, for any given L and amount of A above the $dA/dt = 0$ line will result in A falling vertically to it or rising toward it from any value of A below that line. The $dA/dt = 0$ locus has a flatter slope than the $dL/dt = 0$, as required by the ‘determinant’ condition for stability. The state variables will move according to the pattern of the arrows in each of the four regions of the input space.

Demographic Shock

An immediate exercise is the impact of a demographic shock such as the bubonic plague. If we start from the equilibrium point (A^*, L^*) in *Figure 5*, the population and hence the labor force as well collapses instantly from L^* to L_0 , while A^* is initially fixed. The real wage rises above w^* while the rent per acre falls below r^* . Hence, the land price also falls below p^* . In terms of *Figure 2* the land-labor ratio moves to the right, to a_0 , so that p falls from p^* along the demand curve to the point corresponding to $p_0(a_0)$. The fall in p and rise in w make the amount of new land cleared fall below μA^* and so the area of land cultivated contracts. The rise in per capita output and consumption stimulates the recovery of population and the labor force so the state variables move down and to the right along the path indicated in *Figure 5*. Once the $dA/dt = 0$ line is crossed from above the rising price of land and the falling real wage result in the new land cleared exceeding the ‘depreciation’ μA , and so the area cultivated rises along with the labor force back to the equilibrium point (L^*, A^*) .

Comparative Steady States

Suppose that there is Hicks-neutral technological progress in the production function (1). At the original land-labor ratio this will raise productivity and hence consumption per capita, so as to increase population growth. To keep $dL/dt = 0$ it is clear that the land-labor ratio a^* has to fall. This implies a rotation to the right of the $dL/dt = 0$ ray as in *Figure 6* below. The $dA/dt = 0$ line remains unchanged and so the new equilibrium point (A^{**}, L^{**}) involves an increase in both the labor force and the area of cultivation, with a lower land-labor ratio. Initially A remains fixed at A^* since both p^* and w^* rise in the same proportion. But the increase in labor lowers the real wage rate and raises the rent and the price of land, and thus induces an expansion in the area cultivated as well.

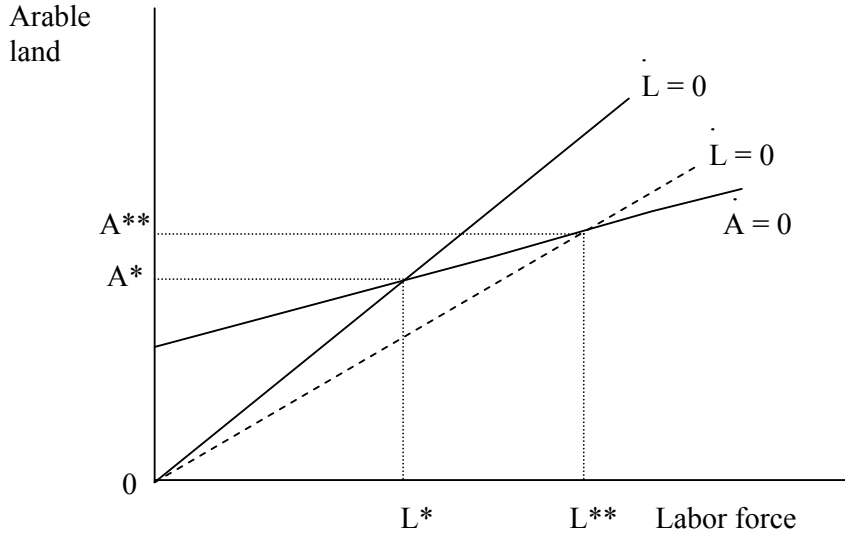


Figure 6: Technological progress

An improvement in the cost of land clearance, due either to better technology or climatic conditions, will shift the $L_a(A)$ function in *Figure 3* downwards for each value of A , and reduce the slope $L_a'(A)$ as well. Thus, for each value of L on the $dA/dt = 0$ line, the area cultivated, A , will have to rise until $p(a)$ has fallen and $w(a)$ has risen sufficiently to equate the price to the marginal cost of land clearance. As we can see from *Figure 7* this will result in A^{**} and L^{**} both higher than the previous values A^* and L^* .

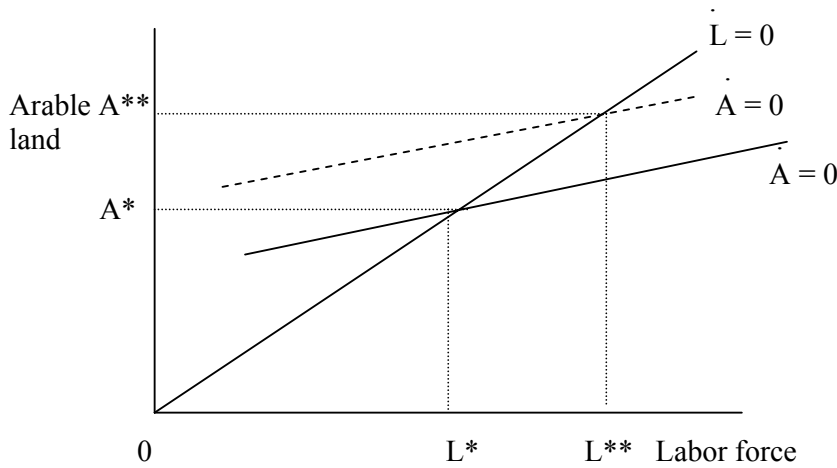


Figure 7: A reduction of the cost of land clearance

From (13) it also follows that a reduction in the rate of time preference δ or the ‘depreciation’ rate μ of land will raise the value of the marginal acre and hence require an increase in A to leave dA/dt still equal to zero for each value of L , while leaving the $dL/dt = 0$ function unchanged. The result is therefore the same as in *Figure 7*.

As a final exercise a leftward shift in the fertility function $f(c)$ will raise c^* and hence y^* and c^* . The result is to rotate the $dL/dt = 0$ locus to the left, which results in a new equilibrium with A^{**} less than A^* and L^{**} less than L^* , as in *Figure 8*.

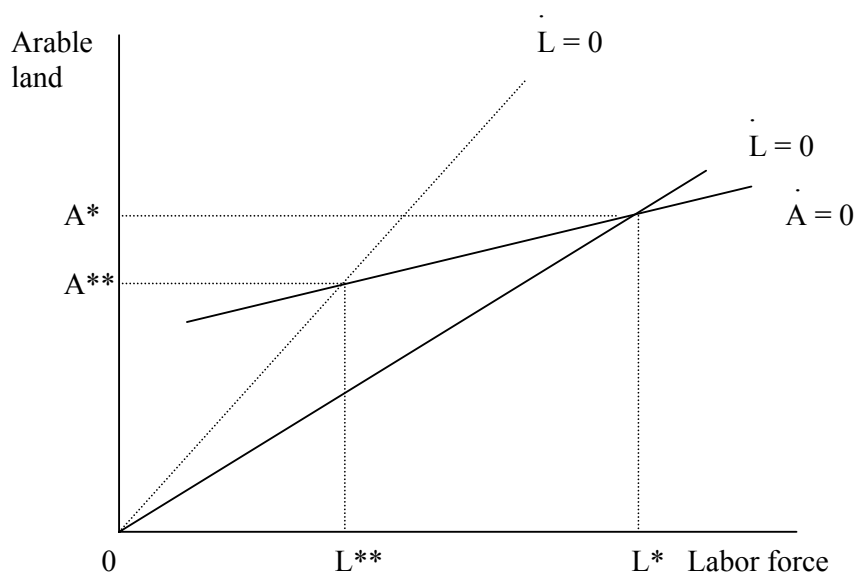


Figure 8: A fertility reduction

Performance: The Byzantine Empire

In the next three sections we will briefly examine the economic performance of the three systems, mainly with respect to population, labor force, cultivated area and output but also looking at other factors such as urbanization and trade as indicators of these developments. The intention is to see whether we can relate these outcomes to the implications of the model and what we know of the exogenous shocks that occurred in each case.

As we have noted the Byzantine Empire before the plague struck in 542 was at its peak in terms of territory and population, put at about 26 million by Treadgold (1997:278) on the basis of estimates by McEvedy and Jones (1980). It is easy to imagine, with Cyril Mango (1994:68-69), what must have happened to the economy:

... all normal occupations were interrupted, prices of goods trebled and quadrupled, starvation set in, fields were deserted and the remaining farmers were burdened with additional taxes on the non-productive land of their deceased neighbours.

There can be little doubt that the plagues of the sixth century, combined with an unprecedented sequence of natural disasters were a factor, perhaps the determining factor in the collapse of urban life. For it is a fact (though some historians still refuse to recognize it) that all round the Mediterranean, the cities, as they had existed in Antiquity, contracted and then practically disappeared.

By 610, as a result of the plague, the Byzantine population had fallen to 17 million, and it continued to fall to a mere 7 million by 780 after the loss of Egypt, Syria, Palestine and North Africa to the Arabs, and further outbreaks of the plague. The population of Constantinople, which was at least 300,000 at the time of Justinian, fell to a mere 40,000 by the middle of the eighth century, after another disastrous outbreak of the plague in 747, 'probably ... the lowest point in the medieval history of Constantinople' (Mango, 1994:78). Antioch and Alexandria, the next largest cities after Constantinople, were lost to the Arabs in the seventh century. Thessalonica was the only other remaining city of any size. Several of the cities of Asia Minor declined to mere *kastra*, citadels surrounded by small civilian settlements (Mango, 1994:Ch. 3, Cameron, 1993:Ch. 7-8)). That this development should not have been related to the appearance of the plague is difficult to believe. Even an author as cautious as Averil Cameron (1993:164) is quite emphatic:

The effects are certainly hard to quantify ... but it is hard not to think that plague must have been a factor in undermining the generally thriving state of cities in the Near East in the early part of the sixth century. [...] Historians vary sharply in the amount of weight that they are willing to attach to the sixth- and seventh-century plague. Yet the fact remains that this seems to have been the first appearance of bubonic plague in Europe; its impact must therefore have been far greater than that of the regular diseases which ravaged ancient cities as a matter of course.

The period 641-780 is described by Treadgold (2002) as the ‘Struggle for Survival’, against the twin enemies of the Arabs and the plague. Byzantium may have escaped total collapse by a slim margin. State revenue fell drastically as people died (Treadgold, 2001:67):

The evident explanation for this decline in state spending is the same as that for most of Justinian’s failures: the effects of the plague after 541. These were so different from most historical events in magnitude and kind that they were inevitably underestimated by most Byzantine observers, as indeed they have been by most modern historians. Not even the return of the plague in the thirteenth century wreaked such havoc, because it recurred less often and reduced a denser population. If the first outbreak had killed Justinian, as it almost did, it might well have brought on the fiscal and military collapse of the empire that he barely averted.

It was bad enough as it was.

While revenue and population shrank, the size of the army did not fall in the same proportion, so a greater share of resources was devoted to the military, and a reorganization of society along military lines, in so-called *themata* (military districts, each with its own army), took place. As our model predicts, however, per capita income appears to have been on the rise and income disparities were being reduced. Treadgold (2002:150) states that ‘ [in] comparison with earlier Byzantine times the rich appear to have been poorer and the poor richer’, which is consistent with the rise in the land-labor ratio lowering rents and raising wages, in terms of our model. He also observes that peasants benefited from the greater availability of land. Labor had become a scarce factor and the peasants benefited from this. It became difficult to tie them to the soil. Their freedom of movement increased and they had ‘complete and unlimited legal disposal over ... [their] land. The Byzantine sources show quite clearly that peasant land was handed down from generation to generation by inheritance and that it could be freely alienated by the possessor just as he chose – by sale, by gift, or limited lease’ (Ostrogorsky, 1966:210). The Byzantine Empire had uncultivated land at its disposal, in its very heartland, Asia Minor, where the military holdings granted out in return for military service could not obtain labor without bringing in foreigners, mostly of Slavonic origin. The rise in per capita income that all this seems to indicate could also explain how a greater than proportionate military

burden was borne, relative to the pre-plague years. In relation to our model the situation would be ripe for the rise in per capita consumption to induce a recovery of population and an associated increase in the area under cultivation.

While the empire had survived, its prospects around the year 800 did not look too good in the ongoing tripartite struggle for power against its two great rivals, the Carolingian Empire of the West and the Abbasid Empire of Haroun al-Rashid in Baghdad. Byzantium had to contend not only with the Arabs but with formidable barbarian peoples on its frontiers, such as the Bulgars and the Pechenegs. In the west the aggressive Normans intruded successfully into Byzantine possessions in southern Italy and the Balkans, after taking Sicily from the Arabs (Norwich, 1991, 1997).

Population, and with it manpower and revenue, began to revive from 7 million in 780 to 12 million in 1025 on the death of the Macedonian Emperor Basil II, who crushed the Bulgars and stabilized the Balkan frontiers. Also significant, particularly for the future, was the conversion to Christianity of the Kievan Rus ruler Vladimir and his marriage to a Byzantine princess to cement a mutually beneficial alliance. Meanwhile the Carolingian Empire was fragmented in the succession struggles between Charlemagne's heirs and the power of the Sunni Abbasid caliphs was challenged by the Shia Fatimid caliphate in Cairo, while Byzantium preserved its political and religious unity.

As the population recovered, the man-land ratio increased, and the relative bargaining power shifted in favor of the larger landholders, who proceeded to buy up peasant property. Efforts on part of the rulers to protect the peasants proved to be of little avail (Ostrogorsky, 1966:221-22):

... after the death of Basil II [in 1025] the long series of these laws came to an end ... For, as even the government regulations of the tenth century, in spite of their extreme severity, had been unable to suppress the buying-up of peasant and military lands, now the passively benevolent attitude of the government meant that the great landowners' capacity for expansion could develop to the full. The destruction of the small freehold properties continued unrestricted; the great landowners absorbed the land of the peasants and soldiers and made the owners their serfs.

[...] Certainly there were free peasants in the late Byzantine period; but whereas in the middle Byzantine period, from the seventh to the beginning of the eleventh century, the free and moving peasantry is the chief factor in agrarian development and the backbone of Byzantine agriculture, from the eleventh century onward, just as in the early period, the great landlord dominates the scene. The agrarian history of the late Byzantine period is that of great landowners and their dependants.

Just when it seemed that the empire could continue its revival a fateful new challenge arose with the emergence of the Seljuk Turks. They took over much of the interior of the Anatolian Plateau after defeating the Byzantines at Manzikert in 1071. This shifted the center of gravity of the empire from the interior of Asia Minor to the coastal regions of the Balkans and the Black Sea. The rising commercial and naval power of Venice, Pisa, Genoa and the other Italian cities was accommodated by the Byzantines with increasing diplomatic concessions, such as exemptions from customs duties and 'extraterritorial' rights in Constantinople (Nicol, 1988). The relative expansion of the West was strikingly displayed in the Crusades, which liberated Jerusalem and Antioch and established new kingdoms in the Holy Land (Runciman, 1951, Riley-Smith, 1987). These Christian allies did not accept the authority of Byzantium, and the Fourth Crusade of 1204 that sacked Constantinople and established the so-called Latin Empire that lasted until 1261 was a fatal wound from which the empire never recovered until its fall at the hands of the Ottoman Turks in 1453 (Queller and Madden, 1997).

Despite the economic expansion from the tenth century onwards, part of which has been well documented by Harvey (1989), it is difficult to resist the impression that the economic history of the Byzantine Empire was one of protracted demographic collapse, due to plague and the Arab invasions, from about 550 to 780, followed by a long recovery during which power systematically shifted in favor of the large landowners, set back by the intrusions of the Fourth Crusade and the Turks. Something akin to a new demographic-economic equilibrium was being restored, within the restricted post-plague area imposed by the Muslim advances, as would be implied by our model. This restoration is symbolized by the fact that Constantinople seems to have had the same population, about 300,000, at the time of Justinian and the onset of the plague, as it had when it was sacked by the Crusaders in 1204, after

having fallen to as low as 40,000 in the middle of the eighth century. This was also about as many inhabitants as it had on the fateful day of 29 May, 1453.

There does not appear to have been any notable technological progress or other economic innovation in the entire history of the empire:

In general, Byzantine technology was extremely conservative. Byzantine agricultural implements remained virtually unchanged from Roman times. The peasant continued to use the light plow dragged by a pair of oxen. It was made of wood and had a removable iron plowshare; it did not have wheels, so the plow bit rather than cut the soil. The scythe was not in use in Byzantium, and the image of Death with its scythe in hand, so popular in the West, would have left the Byzantines unmoved (Kazhdan and Epstein, 1985:27)

The large estates remained partly uncultivated, very much due it seems to faulty techniques (Ostrogorsky, 1966:211):

The difficulty of making proper use of the larger estates was partly due to the primitive conditions of economic technique; for in this respect the Byzantine Empire, so far ahead in culture, was in many ways far behind the West. Thus Byzantium to the end of its days continued to employ an extremely uneconomic and antiquated harness for draught animals, while by the tenth century the West had evolved a greatly improved method of harnessing ...

Byzantium maintained and adopted the administrative, military and economic institutions that it had inherited from antiquity, but did not make any significant new breakthrough, as far as one can gather, other than ‘Greek fire’, the lighted naphtha that was used successfully in naval battles against the Arabs in an early form of chemical warfare, and the importation of the silkworm from China in the sixth century in an early example of commercial espionage. Characteristically, this led to a significant but ‘luxury’ industry, which was also imitated and supplanted in due course by both of its great rivals, the Muslims and the Latins. The critical ‘frontier’, in the case of Byzantium, was always a political and military one, not one of settlement and cultivation. As Harvey (1989) points out, there was considerable land reclamation accompanying the population growth from 900 to 1200 but this is not likely to have constituted any net addition to the cultivated area in the same regions before the onset of the plague. None of this should be taken as denigrating the great resilience and adaptability that the empire displayed in dealing with an almost uninterrupted

succession of military and political challenges and the crucial role in world history that it played in relation to the Russians and the other Slavic peoples of eastern Europe.

The Islamic World

The extent and rapidity of the Arab conquests of the seventh and eighth centuries is one of the most remarkable features of world history. ‘The speed, magnitude, extent and permanence of these conquests excite our wonder and almost affront our reason, but the historian who seeks to explain them is impeded by the deficiency of the evidence at his disposal’, writes J.J. Saunders (1965:39). The first conquests, however, can perhaps be partly explained with the aid of the Findlay (1996) model of the extension of empires, or simply by interpreting the extension of the land frontier in the present model as a military operation. Justinian’s attempt to reconquer the territories lost by Rome to the barbarians had turned the Byzantine frontier into a triple war frontier: Italy, Africa and the east, and the eastern frontier was weakened by the necessity to maintain troops in the two other war theaters.

The eastern war led to nothing and the plague outbursts in the sixth and seventh centuries in the eastern Mediterranean and the western part of the Sassanid Empire left the two contending empires militarily weakened, with little power of resistance, whereas the Arabs had escaped the deadly disease and could take advantage of the population vacuum. Damascus, Jerusalem and Alexandria fell to Islam in rapid succession between 635 and 642, and further east the Arabs simultaneously penetrated Sassanid territory.

Islam, and Arabic, the language in which the Qur’ān was written, imposed a cultural unity over a vast extent of diverse peoples and physical environments, stretching from the shores of the Atlantic to the oases of Central Asia and the mountains of Afghanistan. This vast territory was never a single politically and administratively unified empire but it did maintain a unique identity as the *Dar al-Islam*, or Abode of Islam, as opposed to the *Dar al-Harb*, or Abode of War, i.e. the infidel. Thus most of the welter of dynastic changes and power shifts that form so much a part of medieval Islamic history can be ignored for purposes of this paper since they mostly took place within the same system. It was only on the frontiers of Spain and in the Mediterranean

islands, before the Crusades, that land and people moved in or out of the Islamic world itself as a result of political conflict. In Central Asia much of the conversion of the nomadic Turkish and later Mongol tribes and states took place peacefully. The Crusader kingdoms in Syria and Palestine were exceptions, but only temporarily, since they were eventually returned to the Islamic fold by Saladin and the Mamluks.

In the case of Islam, the extension of the frontier of cultivation was a military operation rather than a peaceful land clearance affair (Hitti:1970:144):

Islam did provide a new battle-cry, a convenient rallying-point and a party watchword. It undoubtedly acted as a cohesive and cementing agency for the heterogeneous masses never before united and furnished a large part of the driving force. But it is hardly in itself enough to explain the conquests. Not fanaticism but economic necessity drove the Bedouin hordes, and most of the armies of conquest were recruited from the Bedouins, beyond the confines of their arid abode to the fair lands of the north. The passion to go to heaven in the next life may have been operative with some, but the desire for the comforts and luxuries of the civilized regions of the Fertile Crescent was just as strong in the case of many.

Fertile soil was available in abundance in the conquered territories, in the valleys of the Guadalquivir, Nile, Euphrates, Tigris, Oxus and Jaxartes rivers, with the Indo-Gangetic plain being added by the Delhi Sultanate early in the thirteenth century. The string of oases from North Africa to Central Asia also yielded high returns. On the whole, however, the Abode of Islam was situated in one of the most arid zones of the civilized world and 'the struggle between the desert and the sown' has been a persistent theme of its existence.

To quote the eminent French geographer Xavier de Planhol (1959:102):

... the heart of Islam remains that desert zone which, slanting across the globe from the Atlantic Ocean to Central Asia, includes the whole of the ancient world between, on the one side, the humid zones of intertropical Africa and monsoon-moistened Asia and on the other side the wet and temperate climate of Europe.

In this zone, according to de Planhol (1959:124) the towns, linked to each other along the trade routes, dominate their rural agrarian environs, which, despite their density, are often no more than 'a farming or truck-gardening suburb'. Reflecting its origins in

Mecca and Medina, Islam in this view is essentially a religion of traders and city-dwellers. It has always had difficulty penetrating mountainous and forested areas. Pastoral nomadism is well adapted to the environmental conditions of the Islamic zone and Bedouin Arab, Berber and Turkish nomadic tribes have all interacted positively and negatively with the sedentary cultivators and townsfolk over the centuries, providing the fourteenth-century Tunisian sage Ibn Khaldun (1958) with the theme of his great work. Interestingly, Khaldun says (1958, Vol.1:302), 'Arabs can only conquer flat territories' and also that (1958, Vol.1:308) 'desert tribes and groups are always dominated by the urban population.'

Jared Diamond (1997) has popularized the idea that agricultural innovations can be diffused more readily across the same latitudes, since climatic conditions are similar, rather than on a north-south axis. With its largely 'horizontal' extent around the globe, within a relatively narrow band, the early Islamic world provides an excellent example of this thesis. As Andrew Watson (1983) has demonstrated convincingly in a seminal work, the first three or four centuries of Islam were marked by a remarkable agricultural revolution that saw a very wide variety of new crops diffused from its eastern margins in India all the way to Morocco and Spain. These included such major crops as cotton and sugarcane, as well as rice, hard wheat, sorghum, citrus fruits, coconuts, bananas, artichoke, spinach and eggplant. The introduction of these crops required complementary efforts in irrigation, the construction of canals and the opening up of new lands. The agents of such change were not only farmers and landowners but also rulers and officials who provided the necessary institutional infrastructure.

The following quotation (Watson, 1983, p.129) describes a process that corresponds exactly to the implications of our model:

... the agricultural revolution was bound up with an ill-documented but none the less real demographic revolution which seems to have touched most parts of the Islamic world from roughly the beginning of the eighth to the tenth century. Rising population levels and increasing levels of output of foodstuffs must continuously have interacted: though both were affected by other factors, at times demographic growth must have been the result of agricultural progress, at times its efficient cause.

Watson cites many instances of this agricultural expansion from all over the Islamic world, and the reader is referred to his book and the many sources cited therein for the evidence. Unfortunately, there appear to be no numbers. Perhaps the strongest evidence for the agricultural revolution of the early Islamic world is the extent and scale of its urbanization. Eric Wolf (1966:13) has defined the peasantry 'in terms of its subordinate relationships to a group of controlling outsiders', and, conversely, urban areas are dependent on the surrounding countryside for their growth.

Watson (1983:132) challenges many published estimates of city size as biased downward, by sheer 'Orientalist' prejudice or 'on the grounds that it is more responsible to underestimate than to overestimate'. Baghdad, and also Samarra, the two capitals of the Abbasid dynasty, could have had populations of close to a million, although half a million would be more plausible and impressive enough, since Constantinople at the time would not have exceeded 300,000. Basra seems to have had at least 200,000. One colorful citation (Watson, 1983:131) says that 'along the Tigris settlement was continuous, so that before dawn crowing cocks answered one another from housetop to housetop all the way from Basra to Baghdad.' On the eve of the Black Death the twin city of Fustat-Cairo also had a population that was of the order of half a million, and Nishapur in the ninth century is put at between 100,000 and five times that number. In the west Fez and Qairawan had several hundred thousand each but Cordoba at its peak was comparable to Baghdad at its height, estimated to be between half and one million. Cordoba also surpassed the one million mark. According to a census taken towards the end of the tenth century, the city had 1,600 mosques, 213,077 houses occupied by the lower and middle classes, another 60,300 inhabited by the higher bureaucracy and the aristocracy, and 80,455 stores (Ocaña Jiménez, 1975:47). The Islamic world also had very many cities of lesser rank, far outstripping Byzantium and Western Europe.

Agriculture clearly had to be very productive to support urban settlements on such a scale, (as well as the Arab cultural advance in general). The relationship was not confined simply to food supply. Major processing industries such as sugar refining and textile manufactures relied on the supply of raw materials from the agricultural sector. In addition to cotton, raw silk and hemp for linen were important inputs to urban textile industries. Many of these manufactures were exported to other parts of

the Islamic world, and also to Europe. In addition to these high value-added manufactured exports, the Islamic world, through the Red Sea and to Persian Gulf, was also the *entrepôt* for the precious spices from India and Southeast Asia to be distributed to Europe.

Looking at the admittedly unsatisfactory data provided by McEvedy and Jones we see that the population of the Islamic world seems to have grown from about 21 million to about 27 million from 700 to 1000, say about 30 percent over three centuries. Issawi (1981) gives 28-36 million for 750 and 35-40 million for 1000. Taking the 28 million low estimate for 750 and the 40 million high estimate for 1000 we get what would be a maximum increase of 43 percent for two and a half centuries, as compared with 30 percent for three. While noticeably better than the performance of Byzantium at about 25 percent over three centuries, even the 43 percent over 250 years can hardly be called a 'demographic revolution'. Thus, while fully agreeing with Watson's picture of complementary agricultural and demographic expansions in the early Islamic world, we should not exaggerate the magnitude of either.

There is little doubt that what held back the further progress of early Islamic agriculture was the unfavorable nature of the physical environment and climatic conditions under which it operated. Once the power of conquest was gone, the character of the frontier changed from one of war to one of land clearance. In the 'struggle between the desert and the sown' the sown had three good centuries, from 700 to 1000, but then the desert seems to have begun its counterattack. Islam was thus hemmed in by its unfortunate geographical circumstances.. As Watson says, there is the possibility that the earlier expansion may have become over-extended, with excessive reliance on irrigation leading to soil erosion. Settlement was not continuous in many areas, raising costs of communication.

Warfare and the breakdown of central authority, and the raids of the ever-present nomads, also shrank the margin of cultivation. The social institutions of the *waqf*, donation of land for religious or charitable purposes, and the *iqta*, a revocable military fief, also were not conducive to sustaining agricultural productivity. The McEvedy-Jones population estimates decline from about 27 million in 1000 to about 23 million in 1300, reflecting the loss of cultivable area and the cessation of innovation. The

impact of the Mongol invasion (Findlay and Lundahl, forthcoming), the sacking of Baghdad in 1258 by Hülegü and the annihilation of the Abbasid Caliphate, was also of course an additional major factor in this regard.

Western Europe

Western Europe at the outset of our period had a Latin Christian core, soon to be flanked by a Muslim Spain in the west, with a pagan fringe in the Scandinavian north and in the east, with the Frisians, Saxons, Balts and Slavs inhabiting the shores of the North Sea and the Baltic and the fertile plains beyond the Elbe, the Oder and the Vistula. By the end of the period the pagan fringe had long since been almost entirely converted and the Muslims in the Iberian peninsula had been pushed back all the way south to the narrow confines of the kingdom of Granada, and driven out of Sicily, Crete and other Mediterranean islands that they had temporarily occupied.

Which were the factors behind this extraordinary advance, which the Roman Empire at its height was not even able to attempt, much less achieve? Lynn White (1962, 1978) had a deceptively simple answer, that could be colloquially phrased as ‘it was the stirrup and the plow, stupid’, which led to his being roundly condemned by Sawyer and Hilton (1963) for the intellectual crime of ‘technical determinism’.

The plow in question was the heavy wheeled plow with an iron coulter and plowshare, drawn by a team of horses or oxen, that cut deep into heavy soil in long straight furrows, turning the soil over with a moldboard. Though apparently known in antiquity it was not employed, being unsuitable for Mediterranean conditions, where the much lighter so-called ‘scratch plow’ prevailed. The plains of northern Europe, however, were ideally suited to the heavier instrument. Horses also proved to be more efficient draft animals than oxen, after the invention of the horse collar multiplied the load that they could pull by up to a factor of five. The wear of the heavy loads on the horse’s hooves required the complementary invention of the nailed horseshoe.

Another innovation, the three-field rotation in place of the more traditional two-field, raised productivity per worker by fifty percent, if additional land were available, clearly providing a stimulus to land reclamation on a vast scale. In the two-field system, half the acreage was sown with winter crops, while the other half was left

fallow. This made sense in the Mediterranean area where rainfall was concentrated to the winter, but in northern Europe, where sowing in the spring could bring another crop, one-third of the area was sown in the fall (wheat or rye), another third in the spring (barley, oats or legumes) and the last third was left fallow. The following year the second field was sown in the fall, etc., until the cycle recommenced. The three-field system both increased the area under cultivation and the range of crops grown. The new system was introduced in France during the eighth century, but took time to spread, and it was not until after 1250 that the speed of diffusion became rapid. Even so, the two systems coexisted during the thirteenth and fourteenth centuries (Grigg, 1980:73).

This new technology required changes in the social organization of the peasant community to adopt a more cooperative basis, in order to take advantage of the economies of scale offered by the more expensive plow and draft animals. The plow is a labor-saving device. It was introduced by a society that had undergone a population decline, but below a certain ‘threshold farm size’ the introduction of the plow is not profitable, and when the cost of feeding the draft animals is taken into account this size increases (Lundahl, 1979:590-91).⁴ The need for oats to feed the horses stimulated greater variety in the mix of crops. These complementarities led to a highly productive system of mixed farming with cereal cultivation and livestock raising becoming the predominant form of agriculture in much of western Europe.

The stirrup, unknown in antiquity despite its apparent simplicity, led to a revolution in warfare with the armored knight on his powerful warhorse relying on the lance to pulverize opposition. The high cost of horse and armor, and the long period of training required for knightly proficiency, would have been impossible without the surplus for rents and taxes over the peasant’s consumption that the heavy plough provided. At a time such as the ninth and tenth centuries, with incessant raids by the Muslims in the south and west, Vikings on the northern coasts and marauding Magyars from the east, the peasant also needed the protection of the knight, so that ‘stirrup and plow’ together were the effective cultural package powering the rise of

⁴ Let us assume that the introduction of the plow saves N man-days per hectare, and that the daily wage rate is w . The annual capital cost (the sum of depreciation and interest, which we assume to be given),

Western Europe during these centuries (Bloch, 1962). Both peasant and warrior needed the consolation and spiritual guidance of the priest, who in turn was supported and protected in his cloister by the peasant and the warrior, together constituting the ‘Three Orders’ of medieval feudal society as depicted by Georges Duby (1980).

Though the concept of the ‘frontier’ is usually associated with Frederick Jackson Turner (1986) and the westward expansion of the United States in the nineteenth century, it is if anything even more applicable to early medieval Europe, as many authors have realized and pointed out in the excellent study edited by Bartlett and Mackay (1989). There clearly was an ‘internal’ economic frontier in already settled areas, between stretches of arable land and the immense forests surrounding them. When reflecting over the data provided by the Domesday Book,⁵ M.M. Postan (1966:549) unequivocally concludes that the ‘Domesday facts clearly denote dense, and hence ancient settlement, the product of at least six centuries of internal colonization.’

The colonization pattern was generalized in Western Europe (North and Thomas, 1973:Ch.5). Colonization took on different shapes. Trees and bushes were cleared in the neighborhood of villages or in the waste further away, contributing to the scattering of fields. New settlements were established in the forests between villages. Coastal areas were reclaimed by the construction of embankments, and marshes were drained. Most important was the advance into forests. During the first centuries after the plague, ‘the forest seems to have held sway over the whole natural landscape’ of Western Europe, as Duby (1974:5) writes. Lowland forest was being cleared in the eleventh and twelfth centuries both in England and France, and at the end of the twelfth century the process continued into upland areas as well, in Brittany, above the Moselle, in the Vosges, the Alps and the Pyrenées (Grigg, 1980:71-72).

There also was an ‘external’ military frontier, which required armed force to push back Muslim warriors in Spain during the entire *Reconquista*, or Saxon and Slav pagan tribes east of the Elbe, particularly after 1150, settling and cultivating the lands

of plowing S hectares is $c = SNw$, and $S = (c/w)(1/N)$ defines the lowest (‘threshold’) farm size that makes it profitable to introduce the plow.

thus won and either converting, expelling or killing the previous occupants. With land abundant and labor scarce, particularly on the eastern frontier, once the Saxons or Slavs accepted German rule, Jesus Christ as the 'Deus Teutonicus' and the German heavy plow in place of their less efficient wooden one, they were valued and protected subjects of the German princes and bishops. After a few generations the population became culturally 'German' despite the abundant and perhaps even dominant presence of Slavic genes. On the other hand Polish and other Christian Slavic rulers also welcomed the efficient German peasants as settlers on their own lands. Assimilation was more difficult in Spain, where *cristianos nuevos* were regarded with considerable suspicion, and even in Ireland where there was tension between the native Celts and the Anglo-Norman feudal lords who were granted estates soon after the conquest.

The complementarity between the stirrup and the plow and the three orders on both the eastern and western frontiers is revealed in the vital role played by the military orders in both cases, the Teutonic Knights in the east and the Orders of Calatrava and Santiago in Spain. The *Reconquista* in Spain, with Santiago Matamoros and El Cid as its foremost symbols, and the wars against the Balts and Slavs, merged into the crusading mentality that swept Christendom, and also saw the excursions against the Albigensian heretics in Provence and into Syria and Palestine against the Muslims under the banner of the cross. Not surprisingly, the movement had permanent effects within the contiguous frontiers on the mainland of Europe but proved ephemeral in the Holy Land.

The role of Christianity in the extension of the frontier was not confined by any means to the inspiration of crusaders. As the greatest landowner in Europe the church took an active part in promoting land clearance and improving agricultural productivity in other ways. It has also been pointed out, by both Lynn White and Georges Duby, that Christianity induced a more instrumental and exploitative view of nature as something purely for the service of man, rather than as being imbued with a spirit of its own that had to be respected and propitiated. The great forests of beech and oak that were cut down to clear land for pasture and tillage were the abode of

⁵ The Domesday Book was put together some time between 1108 and 1109 or between 1111 and 1113 (Poole, 1955:1, note).

spirits that the animistic pagans worshipped. This cultural shift was thus no less important than the heavy plow or the three-field rotation in making the agricultural revolution possible, so the charge of 'technical determinism' by White's critics is not entirely fair. Indeed most of their objections are on matters of the speed and scope of the technical changes and not on their ultimate significance. Hilton (Sawyer and Hilton, 1963) says that rather than the causal sequence from plow and three-field rotation to food supply to population it might be the other way around, from exogenous increase in population to the need for more food and hence to the changes in agricultural technique, anticipating a thesis advanced by Ester Boserup (1965).

One advantage of the modeling approach adopted in this paper is that it enables us to escape from being trapped in 'chicken-egg' arguments of this type. In our model both food supply and population are endogenous variables, depending upon the state of technical knowledge, geographic and climatic conditions and the biological and behavioral determinants of fertility and mortality rates. The heavy plow may have been known to antiquity, and for that matter to the Islamic world, but there was no incentive to adopt it under Mediterranean environmental and social conditions, as White himself clearly says.

The results in terms of population growth in any case were spectacular, as seen from the figures already cited. The McEvedy-Jones numbers further indicate that growth was particularly rapid in the areas of modern Germany and Poland, where it trebled over the period from 1000 to 1340 and also in the areas of modern France and Great Britain. Italy doubled from 5 to 10 million over the same period but even this was below the average for Europe as a whole. These patterns of population growth in the 'old' and 'new' areas of Europe are consistent with the 'frontier thesis' adopted in this paper.

Hand in hand with the increase in area cultivated and population went an increase in the growth of towns and trade. While in 1000 there were only a hundred places in Europe that could be called towns, and half of them were in Italy, three hundred years later the figure had increased to between 4,000 and 5,000 (Grigg, 1980:77). Western European cities, however, were far smaller than their Islamic counterparts. In 1292, Paris had a population of no more than 59,000 people, and as of 1328 the figure had

possibly increased to 80,000. Toulouse, an example of a large provincial town, around the same time presumably had no more than 24,000 inhabitants. In Italy, where the largest cities were found, Milan had 52,000 in the thirteenth century, Padua 41,000 in 1320, Naples 27,000 in 1278, Venice 78,000 in 1363, Bologna 32,000 in 1371, Florence 55,000 in 1381. Before the middle of the fourteenth century only Milan, Venice, Naples and Florence, possibly also Palermo, exceeded 50,000 inhabitants. Some of these cities may have approached the 100,000 mark. In the Low Countries, Ghent had some 56,000-60,000 inhabitants in 1356, and Bruges 25,000-35,000 in 1340. Antwerp was a small-town of 18,000 in 1374. In England, London dominated completely, with 35,000-45,000 in 1377 followed by the far smaller York and Bristol, with figures somewhere between 10,000 and 14,000. In Spain, Barcelona, Cordoba and Seville (plus Granada in the Moorish zone) were the only cities with more than 40,000 inhabitants. Only Cologne in Germany was close to the 40,000 mark, while Metz, Strasbourg, Nuremberg, Augsburg, Vienna, Prague, Lübeck, and Magdeburg had populations around 20,000 (Russell, 1958, Tables 63-65, van Werveke, 1963:38-39). Even with a generous allowance for the influence of the Black Death on the later figures the European cities were thus extremely small, relative to Constantinople and the Islamic world.

Trade took place not only within Europe but with the Byzantine and Islamic worlds as well, leading to the well-known identification by Robert Lopez (1976) of a 'commercial revolution' of the thirteenth century that was also augmented by the growing trade with Asia both by sea and overland due to the Pax Mongolica. As is well known a Genoese ship from the Black Sea port of Kaffa engaged in this lucrative trade returned to the Mediterranean in 1347 and opened another act in the drama of 'Rats, Lice and History' (Zinsser, 1935).

Even before the onset of the Black Death, however, the preceding century was marked by what Archibald Lewis (1958) termed 'The Closing of the Medieval Frontier 1250-1350'. The internal frontier was reaching its natural limits within the prevailing technology and there is the possibility that climate may have started to become less favorable. Tillage competed with pasture to the detriment of the proper balance between the two. Population continued to grow while the supply of land was not keeping pace, lowering wages and peasant incomes while raising rents (North and

Thomas, 1973:48), leading to growing inequality and social conflicts in both the countryside and the towns. The external frontiers of Europe also ceased to expand, with the Muslims recovering the Holy Land and the Byzantines Constantinople in 1261.

The second decade of the fourteenth century saw the outbreak of what William Jordan (1996) has called the 'Great Famine' of 1315-22. A population of Europe in excess of 70 millions, under the prevailing technology, was getting to be unsustainable within its geographical confines. A Malthusian crisis of major proportions was clearly looming. Population densities in certain rural areas in 1300 had increased to the point where they were comparable with those of the early nineteenth century, when the farming technology was vastly superior (Grigg, 1980:Ch. 7). David Grigg (1980:82) summarizes:

For perhaps two centuries the expansion of the cultivated area and the adoption of new techniques was sufficient to keep production up with population growth. But by the middle of the thirteenth century the supply of agricultural land was running out; it was in this century that there are most signs of attempts to intensify production by growing legumes and reducing the fallow but they seem to have been insufficient. The primary blockage to improving yields was the lack of livestock manure, and in the densely populated arable areas of south-eastern England and northern France this reflected the lack of grazing, as population growth led to the ploughing of grazing land.

Western European agriculture was getting into a situation typical of a number of today's developing countries, where the pressure of population leads to increasingly intensive cultivation, with diminishing returns to labor and falling productivity per acre, tantamount to a reduction of the effective land area, in a sequence that easily feeds itself (Lundahl, 1979:Ch.5). The Genoese ship from Kaffa was in the nature of a historically necessary accident waiting to happen.

Conclusions

In the middle of the sixth century A.D. the Mediterranean world was struck by the so-called Plague of Justinian, an epidemic that would recur in further successive waves until the mid-eighth century. No good estimates exist with respect to death tolls, but there is not the slightest doubt that in the areas where it hit, the consequences were extremely severe. The diffusion in space was limited, however. Northern and

northwestern Europe apparently escaped and so did the areas that a century later would constitute the core areas of the new world religion: Islam.

As a result of the plague the population of the Eastern Roman Empire hit a trough some time around the late eighth century and then began to rise until the advent of the Black Death in the mid-fourteenth century. Simultaneously, the Islamic population rose, until the Mongol invasions in the mid- thirteenth century. Europe, in turn, saw a population decline in the wake of the plague, followed by a rise that accelerated after 1000 until the early fourteenth century, when famine was followed by another outbreak of the plague. Thus, all three geographical territories display a common demographic pattern: a decline in the size of the population, followed by a rise, continuing on beyond the previous peak.

As our essay demonstrates, this pattern can be made subject to analytical representation in the form of a simple Malthusian model where birth and death rates are functions of per capita consumption, production is a function of labor and land, and the extent of the arable area determined by the existence of an agrarian or military frontier that can be extended at a rising cost in terms of labor.

This simple model can be used to investigate the impact of a demographic shock. The shock leads to an instantaneous rise in the land-labor ratio due to the collapse of population, higher wages and lower rents, and a gradual reduction of the area under cultivation. Per capita consumption, among the survivors, increases and stimulates population growth. Due to the Malthusian characteristics of the model, however, this increase cannot continue, so the land-labor ratio must fall. The price of land increases and induces land clearance at the frontier and an expansion of the area under cultivation until the original levels of both land and labor are restored.

The model can also be used to investigate the effects of a discrete technological change. This raises labor productivity and consumption per capita and so makes the population grow. Population growth, however, cannot go on indefinitely, and in order to make it cease the land-labor ratio must fall. The larger population raises rents and the price of land, inducing an increase of the area under cultivation but less than proportionately to the increase in population, thus bringing about the fall in the land-

labor ratio required to restore the Malthusian equilibrium. The extent of the resulting increases in population and the supply of land depend upon the magnitude of the technical change and the elasticity of the endogenous land frontier with respect to the rise in the price of land that it induces.

A reduction of the cost of land clearance, as a result of technological progress or more favorable climatic conditions, reduces the amount of labor necessary to sustain a given land area as well as the labor needed to clear an acre of new land. The area under cultivation has to increase until the land price has fallen enough and the wage rate risen enough to equate the land price and the marginal cost of land clearance. This, in turn, also increases the population and the labor force, in the same proportion as the increase in the supply of land, to preserve the Malthusian equilibrium.

Analogously, decreases in the rate of time preference or the exogenous rate of land destruction increase the price of land at the margin and hence also the area under cultivation and the labor force in the same proportion.

Finally, an autonomous fertility reduction leads to higher per capita consumption and production in the Malthusian equilibrium. The labor force shrinks, and with it also the area under cultivation, but less than in the same proportion, permitting the economy to enjoy a permanently higher standard of living as a result of the greater relative land-abundance induced by the exogenous reduction in fertility.

The model thus makes some explicit predictions, notably with respect to the demographic shock and the effects of technological change. These predictions can be checked against the historical performance of our three geographical areas: Byzantium, the Islamic world and Western Europe.

In Byzantium the plague (and the subsequent invasions by the Arabs) led to a contraction of the economy. The area under cultivation was reduced and as a result urban life underwent a decline as well. Per capita income, however, increased as the land-labor ratio rose, which made it possible to shoulder a higher military burden per capita. The increase in per capita income induced population growth from some time between the late eighth century and the early eleventh, a process that also received

some aid from the more peaceful conditions prevailing on the political frontier. As predicted by our model, when the population began to grow back, bargaining power in the labor and land markets shifted in favor of the landowners, and this resulted in harsher contractual conditions for the peasantry. Land rents increased at the expense of wages. At the same time, political conditions deteriorated, with the military frontier being pushed in by the Seljuk Turks and the Fourth Crusade. This opened the door for the eventual collapse of the empire with the fall of Constantinople in 1453. Nowhere in this process does technological progress appear to have been present, so that the Byzantine story was simply one of population change and military operations within a shrinking territory and a concomitant loss of population to superior military force.

The core territory of what would later become the Abode of Islam escaped the ravages of the plague. The cycle began by territorial expansion. The military frontier between the Byzantine and Sassanid empires was weakened by the loss of population due to the plague, and into this population vacuum the Arabs could move relatively easily, pushing both north and west, into Byzantine territory in northern Africa, and also east into Iran all the way to the region of Sind in the west of India. The area of arable land was extended not by clearance but by conquest. Territorial expansion was, however, accompanied by remarkable technological progress in agriculture. New crops stimulated the use of irrigation and land clearance, industrial processing of agricultural products, and also demographic growth and urban development. The population of the Islamic world would grow from some time between 700 and 750 to 1000 at a rate that was superior to that of Byzantium, but not one that could be called a 'demographic revolution'. Such a revolution was not possible under the unfavorable conditions of the physical environment. The largely desert-like conditions in much of the territory of Islam could not support a higher population. To this can be added the unfortunate impact of certain social institutions and the devastating blow dealt to Islam by the Mongols.

The last of our three cases, Western Europe, emerges by a wide margin as the 'winner' in our imaginary 'population race'. Of course, we can only speculate about the reasons for this, in the light of our model. One factor was technological change: the improvement of the plow, the substitution of the horse for the ox, the horse collar, the horseshoe, the three-field system of crop rotation and the change in the crop mix,

as well as the concomitant changes in the social system. More important, however, was what took place at the frontier of cultivation. In different ways the frontier was spectacularly extended. The dense forests had to yield to the sown. While in the case of the Islamic world the desert increasingly encroached on the sown, in Western Europe population growth induced by technical progress ensured the retreat of the wilderness. The Germans advanced towards the east across the Elbe in an early *Drang nach Osten*, and the Spaniards pushed back the Moors, in an effort that would carry over to overseas conquests beginning in 1492, the very year that the military frontier was closed in the Iberian Peninsula itself. The results of all these frontier movements were spectacular, far above the achievements by Byzantium and Islam. Cities grew, albeit not to the size of those of Byzantium and Islam, but grow they did, both in size and in numbers. Western Europe experienced the same cycle as the other two regions, first a downswing, then an upturn in population. The upturn was nothing short of spectacular in the Malthusian age that we are dealing with, to the point where at the end of the period symptoms of serious overpopulation began to appear. The advent of the Black Death drastically raised the land-labor ratio back to a much more favorable level. In a perverse sense it came as a deliverer, triggering a new demographic-economic cycle that would last all the way up to the Industrial Revolution (Findlay and Lundahl, 2002).

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